

# 2-6

## Proving Angles Congruent

### Common Core State Standards

**G-CO.C.9** Prove theorems about lines and angles.  
Theorems include: . . . vertical angles are congruent . . .

**MP 1, MP 3, MP 4, MP 6**

**Objective** To prove and apply theorems about angles



Use what you've learned about congruent angle pairs.



Getting Ready!

A quilter wants to duplicate this quilt but knows the measure of only two angles. What are the measures of angles 1, 2, 3, and 4? How do you know?

### Lesson Vocabulary

- theorem
- paragraph proof

In the Solve It, you may have noticed a relationship between vertical angles. You can prove that this relationship is always true using deductive reasoning. A **theorem** is a conjecture or statement that you prove true.

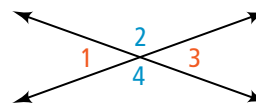
**Essential Understanding** You can use given information, definitions, properties, postulates, and previously proven theorems as reasons in a proof.

take note

### Theorem 2-1 Vertical Angles Theorem

Vertical angles are congruent.

$$\angle 1 \cong \angle 3 \text{ and } \angle 2 \cong \angle 4$$

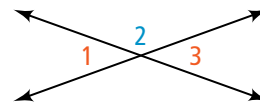


When you are writing a geometric proof, it may help to separate the theorem you want to prove into a **hypothesis** and **conclusion**. Another way to write the Vertical Angles Theorem is “**If two angles are vertical, then they are congruent.**” The hypothesis becomes the given statement, and the conclusion becomes what you want to prove. A two-column proof of the Vertical Angles Theorem follows.

## Proof of Theorem 2-1: Vertical Angles Theorem

**Given:**  $\angle 1$  and  $\angle 3$  are vertical angles.

**Prove:**  $\angle 1 \cong \angle 3$



Statements	Reasons
1) $\angle 1$ and $\angle 3$ are vertical angles.	1) Given
2) $\angle 1$ and $\angle 2$ are supplementary. $\angle 2$ and $\angle 3$ are supplementary.	2) $\angle$ s that form a linear pair are supplementary.
3) $m\angle 1 + m\angle 2 = 180$ $m\angle 2 + m\angle 3 = 180$	3) The sum of the measures of supplementary $\angle$ s is 180.
4) $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	4) Transitive Property of Equality
5) $m\angle 1 = m\angle 3$	5) Subtraction Property of Equality
6) $\angle 1 \cong \angle 3$	6) $\angle$ s with the same measure are $\cong$ .

### Plan

How do you get started?

Look for a relationship in the diagram that allows you to write an equation with the variable.



### Problem 1 Using the Vertical Angles Theorem

### GRIDDED RESPONSE

What is the value of  $x$ ?

#### Think

The two labeled angles are vertical angles, so set them equal.

Solve for  $x$  by subtracting  $2x$  from each side and then dividing by 2.

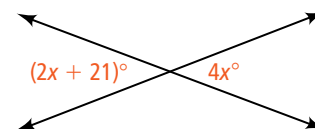
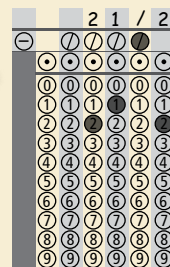
Grid the answer as  $21/2$  or 10.5.

#### Write

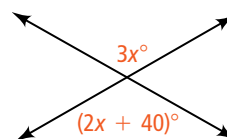
$$2x + 21 = 4x$$

$$21 = 2x$$

$$\frac{21}{2} = x$$



**Got It?** 1. What is the value of  $x$ ?

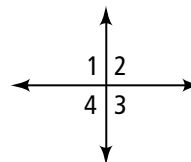




## Problem 2 Proof Using the Vertical Angles Theorem

**Given:**  $\angle 1 \cong \angle 4$

**Prove:**  $\angle 2 \cong \angle 3$



Statements	Reasons
1) $\angle 1 \cong \angle 4$	1) Given
2) $\angle 4 \cong \angle 2$	2) Vertical angles are $\cong$ .
3) $\angle 1 \cong \angle 2$	3) Transitive Property of Congruence
4) $\angle 1 \cong \angle 3$	4) Vertical angles are $\cong$ .
5) $\angle 2 \cong \angle 3$	5) Transitive Property of Congruence

### Think

**Why does the Transitive Property work for statements 3 and 5?**

In each case, an angle is congruent to two other angles, so the two angles are congruent to each other.

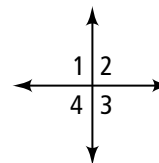


**Got It?** 2. a. Use the Vertical Angles Theorem to prove the following.

**Given:**  $\angle 1 \cong \angle 2$

**Prove:**  $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$

b. **Reasoning** How can you prove  $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$  without using the Vertical Angles Theorem? Explain.

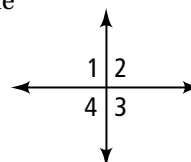


The proof in Problem 2 is two-column, but there are many ways to display a proof. A **paragraph proof** is written as sentences in a paragraph. Below is the proof from Problem 2 in paragraph form. Each statement in the Problem 2 proof is red in the paragraph proof.

**Proof Given:**  $\angle 1 \cong \angle 4$

**Prove:**  $\angle 2 \cong \angle 3$

**Proof:**  $\angle 1 \cong \angle 4$  is given.  $\angle 4 \cong \angle 2$  because vertical angles are congruent. By the Transitive Property of Congruence,  $\angle 1 \cong \angle 2$ .  $\angle 1 \cong \angle 3$  because vertical angles are congruent. By the Transitive Property of Congruence,  $\angle 2 \cong \angle 3$ .



The Vertical Angles Theorem is a special case of the following theorem.



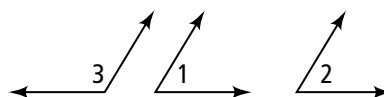
### Theorem 2-2 Congruent Supplements Theorem

#### Theorem

If two angles are supplements of the same angle (or of congruent angles), then the two angles are congruent.

#### If ...

$\angle 1$  and  $\angle 3$  are supplements and  $\angle 2$  and  $\angle 3$  are supplements



#### Then ...

$\angle 1 \cong \angle 2$

You will prove Theorem 2-2 in Problem 3.

## Plan

How can you use the given information?

Both  $\angle 1$  and  $\angle 2$  are supplementary to  $\angle 3$ . Use their relationship with  $\angle 3$  to relate  $\angle 1$  and  $\angle 2$  to each other.

Proof

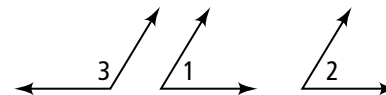


### Problem 3 Writing a Paragraph Proof

**Given:**  $\angle 1$  and  $\angle 3$  are supplementary.  
 $\angle 2$  and  $\angle 3$  are supplementary.

**Prove:**  $\angle 1 \cong \angle 2$

**Proof:**  $\angle 1$  and  $\angle 3$  are supplementary because it is given. So  $m\angle 1 + m\angle 3 = 180$  by the definition of supplementary angles.  $\angle 2$  and  $\angle 3$  are supplementary because it is given, so  $m\angle 2 + m\angle 3 = 180$  by the same definition. By the Transitive Property of Equality,  $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$ . Subtract  $m\angle 3$  from each side. By the Subtraction Property of Equality,  $m\angle 1 = m\angle 2$ . Angles with the same measure are congruent, so  $\angle 1 \cong \angle 2$ .



**Got It?** 3. Write a paragraph proof for the Vertical Angles Theorem.

The following theorems are similar to the Congruent Supplements Theorem.

Take note

### Theorem 2-3 Congruent Complements Theorem

#### Theorem

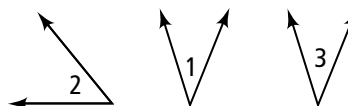
If two angles are complements of the same angle (or of congruent angles), then the two angles are congruent.

If ...

$\angle 1$  and  $\angle 2$  are complements  
 and  $\angle 3$  and  $\angle 2$  are  
 complements

Then ...

$\angle 1 \cong \angle 3$



You will prove Theorem 2-3 in Exercise 13.

### Theorem 2-4

#### Theorem

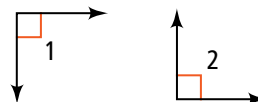
All right angles are congruent.

If ...

$\angle 1$  and  $\angle 2$  are right angles

Then ...

$\angle 1 \cong \angle 2$



You will prove Theorem 2-4 in Exercise 18.

### Theorem 2-5

#### Theorem

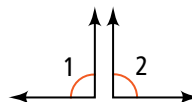
If two angles are congruent and supplementary, then each is a right angle.

If ...

$\angle 1 \cong \angle 2$ , and  $\angle 1$  and  $\angle 2$   
 are supplements

Then ...

$m\angle 1 = m\angle 2 = 90$



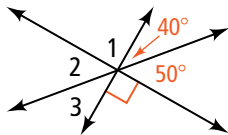
You will prove Theorem 2-5 in Exercise 23.



## Lesson Check

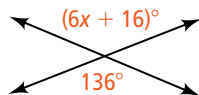
### Do you know HOW?

1. What are the measures of  $\angle 1$ ,  $\angle 2$ , and  $\angle 3$ ?



2. What is the value of  $x$ ?

- (A) 12      (C) 120  
(B) 20      (D) 136



### Do you UNDERSTAND?



3. **Reasoning** If  $\angle A$  and  $\angle B$  are supplements, and  $\angle A$  and  $\angle C$  are supplements, what can you conclude about  $\angle B$  and  $\angle C$ ? Explain.
4. **Error Analysis** Your friend knows that  $\angle 1$  and  $\angle 2$  are complementary and that  $\angle 1$  and  $\angle 3$  are complementary. He concludes that  $\angle 2$  and  $\angle 3$  must be complementary. What is his error in reasoning?
5. **Compare and Contrast** How is a theorem different from a postulate?

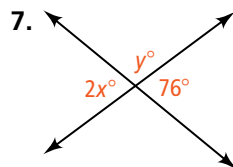
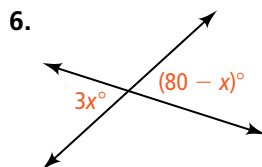


## Practice and Problem-Solving Exercises



### A Practice

Find the value of each variable.



8.

Find the measures of the labeled angles in each exercise.

9. Exercise 6

10. Exercise 7

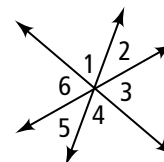
11. Exercise 8

12. **Developing Proof** Complete the following proof by filling in the blanks.

**Given:**  $\angle 1 \cong \angle 3$

**Prove:**  $\angle 6 \cong \angle 4$

Statements	Reasons
1) $\angle 1 \cong \angle 3$	1) Given
2) $\angle 3 \cong \angle 6$	2) a. ?
3) b. ?	3) Transitive Property of Congruence
4) $\angle 1 \cong \angle 4$	4) c. ?
5) $\angle 6 \cong \angle 4$	5) d. ?



See Problem 2.

13. **Developing Proof** Fill in the blanks to complete this proof of the Congruent Complements Theorem (Theorem 2-3).

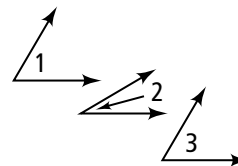
See Problem 3.

If two angles are complements of the same angle, then the two angles are congruent.

**Given:**  $\angle 1$  and  $\angle 2$  are complementary.  
 $\angle 3$  and  $\angle 2$  are complementary.

**Prove:**  $\angle 1 \cong \angle 3$

**Proof:**  $\angle 1$  and  $\angle 2$  are complementary and  $\angle 3$  and  $\angle 2$  are complementary because it is given. By the definition of complementary angles,  $m\angle 1 + m\angle 2 = \mathbf{a. \text{ ? }}$  and  $m\angle 3 + m\angle 2 = \mathbf{b. \text{ ? }}$ . Then  $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$  by the Transitive Property of Equality. Subtract  $m\angle 2$  from each side. By the Subtraction Property of Equality, you get  $m\angle 1 = \mathbf{c. \text{ ? }}$ . Angles with the same measure are  $\mathbf{d. \text{ ? }}$ , so  $\angle 1 \cong \angle 3$ .



**B Apply**

14. **Think About a Plan** What is the measure of the angle formed by Park St. and 116th St.?

- Can you make a connection between the angle you need to find and the labeled angle?
- How are angles that form a right angle related?



15. **Open-Ended** Give an example of vertical angles in your home or classroom.

**Algebra** Find the value of each variable and the measure of each labeled angle.

16. 
 $(x + 10)^\circ$  and  $(4x - 35)^\circ$

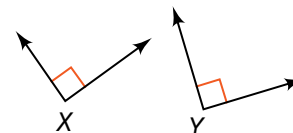
17. 
 $(3x + 8)^\circ$ ,  $(5x - 20)^\circ$ , and  $(5x + 4y)^\circ$

18. **Developing Proof** Fill in the blanks to complete this proof of Theorem 2-4.  
 All right angles are congruent.

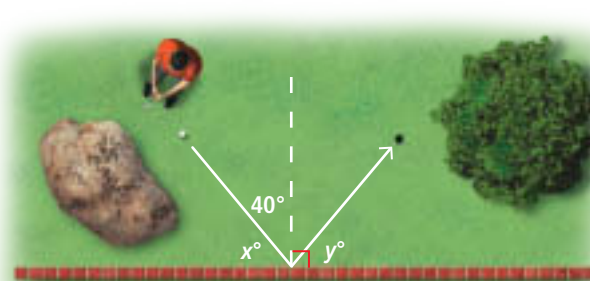
**Given:**  $\angle X$  and  $\angle Y$  are right angles.

**Prove:**  $\angle X \cong \angle Y$

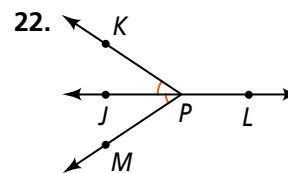
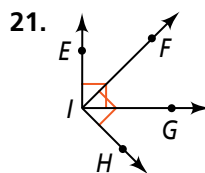
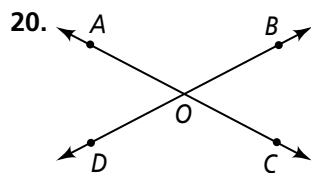
**Proof:**  $\angle X$  and  $\mathbf{a. \text{ ? }}$  are right angles because it is given.  
 By the definition of  $\mathbf{b. \text{ ? }}$ ,  $m\angle X = 90$  and  $m\angle Y = 90$ .  
 By the Transitive Property of Equality,  $m\angle X = \mathbf{c. \text{ ? }}$ .  
 Because angles of equal measure are congruent,  $\mathbf{d. \text{ ? }}$ .



19. **Miniature Golf** In the game of miniature golf, the ball bounces off the wall at the same angle it hit the wall. (This is the angle formed by the path of the ball and the line perpendicular to the wall at the point of contact.) In the diagram, the ball hits the wall at a  $40^\circ$  angle. Using Theorem 2-3, what are the values of  $x$  and  $y$ ?



Name two pairs of congruent angles in each figure. Justify your answers.



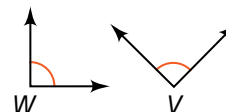
**23. Developing Proof** Fill in the blanks to complete this proof of Theorem 2-5.

If two angles are congruent and supplementary, then each is a right angle.

**Given:**  $\angle W$  and  $\angle V$  are congruent and supplementary.

**Prove:**  $\angle W$  and  $\angle V$  are right angles.

**Proof:**  $\angle W$  and  $\angle V$  are congruent because **a. ?**. Because congruent angles have the same measure,  $m\angle W =$  **b. ?**.  $\angle W$  and  $\angle V$  are supplementary because it is given. By the definition of supplementary angles,  $m\angle W + m\angle V =$  **c. ?**. Substituting  $m\angle W$  for  $m\angle V$ , you get  $m\angle W + m\angle W = 180$ , or  $2m\angle W = 180$ . By the **d. ?** Property of Equality,  $m\angle W = 90$ . Since  $m\angle W = m\angle V$ ,  $m\angle V = 90$  by the Transitive Property of Equality. Both angles are **e. ?** angles by the definition of right angles.



**24. Design** In the photograph, the legs of the table are constructed so that  $\angle 1 \cong \angle 2$ . What theorem can you use to justify the statement that  $\angle 3 \cong \angle 4$ ?

**25. Reasoning** Explain why this statement is true: If  $m\angle ABC + m\angle XYZ = 180$  and  $\angle ABC \cong \angle XYZ$ , then  $\angle ABC$  and  $\angle XYZ$  are right angles.

**Algebra** Find the measure of each angle.

**26.**  $\angle A$  is twice as large as its complement,  $\angle B$ .

**27.**  $\angle A$  is half as large as its complement,  $\angle B$ .

**28.**  $\angle A$  is twice as large as its supplement,  $\angle B$ .

**29.**  $\angle A$  is half as large as twice its supplement,  $\angle B$ .

**30.** Write a proof for this form of Theorem 2-2.

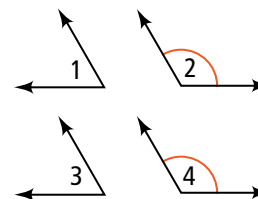
**Proof** If two angles are supplements of congruent angles, then the two angles are congruent.

**Given:**  $\angle 1$  and  $\angle 2$  are supplementary.

$\angle 3$  and  $\angle 4$  are supplementary.

$\angle 2 \cong \angle 4$

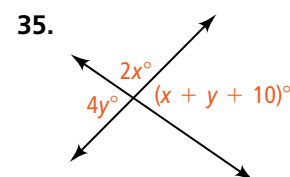
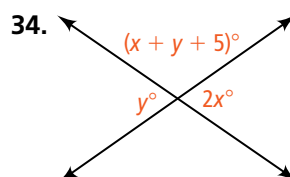
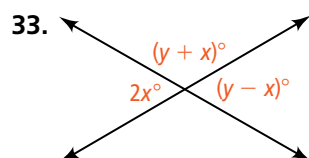
**Prove:**  $\angle 1 \cong \angle 3$



**31. Coordinate Geometry**  $\angle DOE$  contains points  $D(2, 3)$ ,  $O(0, 0)$ , and  $E(5, 1)$ . Find the coordinates of a point  $F$  so that  $\overline{OF}$  is a side of an angle that is adjacent and supplementary to  $\angle DOE$ .

32. **Coordinate Geometry**  $\angle AOX$  contains points  $A(1, 3)$ ,  $O(0, 0)$ , and  $X(4, 0)$ .
- Find the coordinates of a point  $B$  so that  $\angle BOA$  and  $\angle AOX$  are adjacent complementary angles.
  - Find the coordinates of a point  $C$  so that  $\overrightarrow{OC}$  is a side of a different angle that is adjacent and complementary to  $\angle AOX$ .

**Algebra** Find the value of each variable and the measure of each angle.

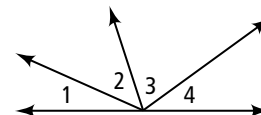


## Standardized Test Prep

### GRIDDED RESPONSE



36.  $\angle 1$  and  $\angle 2$  are vertical angles. If  $m\angle 1 = 63$  and  $m\angle 2 = 4x - 9$ , what is the value of  $x$ ?
37. What is the area in square centimeters of a triangle with a base of 5 cm and a height of 8 cm?
38. In the figure at the right,  $m\angle 1 = \frac{1}{2}(m\angle 2)$ ,  $m\angle 2 = \frac{2}{3}(m\angle 3)$ . If  $m\angle 3 = 72$ , what is  $m\angle 4$ ?
39. What is the measure of an angle with a supplement that is four times its complement?



## Mixed Review

Which property of equality or congruence justifies going from the first statement to the second?

← See Lesson 2-5.

40.  $3x + 7 = 19$   
 $3x = 12$

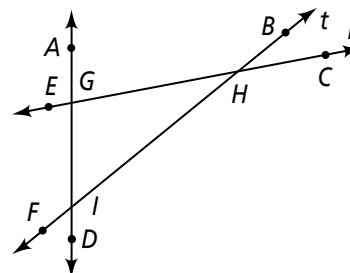
41.  $4x = 20$   
 $x = 5$

42.  $\angle 1 \cong \angle 2$  and  $\angle 3 \cong \angle 2$   
 $\angle 1 \cong \angle 3$

**Get Ready!** To prepare for Lesson 3-1, do Exercises 43–48.

Refer to the figure at the right.

- Name four points on line  $t$ .
- Are points  $G$ ,  $A$ , and  $B$  collinear?
- Are points  $F$ ,  $I$ , and  $H$  collinear?
- Name the line on which point  $E$  lies.
- Name line  $t$  in three other ways.
- Name the point at which lines  $t$  and  $r$  intersect.



← See Lesson 1-2.